THEORY OF COMPUTATION (TOC)

TUTORIAL -1

Prove the following by Mathematical induction:

1. The sum of first n natural numbers, $1 + 2 + 3 + \dots + n$ is $\frac{n(n+1)}{2}$
2. The sum of first n positive odd numbers, $1 + 3 + 5 + \dots + (2n-1)$ is n^2
3. The sum of first n positive even numbers, $2 + 4 + 6 + \dots + 2n$ is $n(n+1)$
4. $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{(n+1)}$
5. Show that $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$ is true for all nonnegative integers $(n \geq 0)$.
6. $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
7. $1\times1!+2\times2!+3\times3!+\dots+n\times n!=(n+1)!-1$ Where $(n\geq1)$
8. $1.2.3 + 2.3.4 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
9. $1^2 - 2^2 + 3^2 - \dots + (-1)^{(n+1)} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}$
10. $n^4 - 4n^2$ is divisible by 3 for all $n\geq0$.
11. For all $n\in N$, $n(n^2 + 5)$ is a multiple of 6. Where $(n\geq0)$.
12. $n < 2^n$ for all positive integers n.
13. $2^n < n!$, where n is any positive integer with $n\geq4$.
14. $5^n - 1$ is divisible by 4 for all integers $n\geq0$.

Prove the following by using Proof by Contradiction:

1. The sum of a rational number and an irrational number is irrational.
2. If a^2 is even, then a is even. (Where $a\in Z$).
3. If 3n+2 is odd, then n is odd.
4. $\sqrt{2}$ is irrational.
5. The function $f(x) = x^5 + 6x^3 + 17x + 1$ cannot have more than one real root.
6. Every nonzero rational number can be expressed as a product of two irrational numbers.

By: SRG@KEC

THEORY OF COMPUTATION (TOC)

TUTORIAL -2

1. Design a Deterministic Finite Automaton (DFA) M that accepts the language L (M) = {w∊ {a, b}*: w has even numbers of b's}.
2. Design a deterministic finite automaton M that accepts the language L (M) = {w∊ {a, b}*: w has abab as substring}.
3. Design a deterministic finite automaton M that accepts the language L (M) = {w∊ {a, b}*: w does not contain three consecutive b's}.
4. Design a deterministic finite state automaton M, which accepts the language L = {w ∊ {0, 1}*: every string w in the language L ends with 00.
5. Design a DFA which accepts the language L = {w ∊ {0, 1}*: second symbol of w is '0' and fourth symbol is '1'}.
6. Design a deterministic finite automaton M that accepts the language L (M) = {w∊ {a, b}*: w has neither aa nor bb as substring}.
7. Design a deterministic finite state automaton M that accepts the language L (M) = {w∊ {0, 1}*: w has even numbers of 0's and odd numbers of 1's}.
8. Construct a DFA that accepts the language L (M) = {w ∊ {0, 1}*: w has number of zeros that are multiple of three}.
9. Design a DFA that accepts the strings over ∑ = {0, 1} that either start with 01 or end with 01.
10. Design a DFA which accepts a set of strings containing four 1's in every string over ∑ = {0, 1}.
11. Design a DFA that accepts the language L (M) = {w∊ {a, b}*: w has number of a's divisible by 3}.
12. Design a DFA that accepts the language L (M) = {w∊ {a, b}*: w has number of a divisible by 3 and number of b by 2}.
13. Design a DFA that accepts the language L (M) = {w∊ {a, b}*: w does not have abb as a substring}.
14. Design a DFA and NFA for the regular expression (abUaba)* with ∑ = {a, b}.
15. Design a NFA for the regular expression (abUba)* with ∑ = {a, b}.
16. Design a NFA for the regular expression (i) (aa)* (ii) aa*ba (iii) (aa)*(bb)*with ∑ = {a, b}.
17. Convert the following NFA to DFA. M=(K, ∑,Δ,s,F) Where K={q_0,q_1,q_2,q_3,q_4}, $\Sigma = \{a, b\}$, s= q_0, F=(q_1,q_4), Δ={(q_0,a,q_0),(q_0,a,q_3),(q_0,∊,q_1),(q_0,b,q_1),(q_1,b,q_2),(q_2,a,q_4),(q_2,b,q_4), (q_3,b,q_2),(q_3,∊ ,q_2), (q_4,a,q_4),(q_4,b,q_4)}.

THEORY OF COMPUTATION (TOC)

TUTORIAL -3

Check whether the following languages are:

a. Regular or not b. Context free or not [Take $\sum = \{a, b\}$ if necessary]

1. $\{a^i b^i : i \geq 0\}$
2. $\{a^i b a^{2i} : i \geq 1\}$
3. $\{a^i b^j : i<j,\ i, j \geq 1\}$
4. $\{a^i b^j : i>j,\ i, j \geq 1\}$
5. $\{a^n b^{2n} : n \geq 1\}$
6. $\{a^p$: p is prime number$\}$
7. $\{a^n$: n is natural number$\}$
8. $\{a^s$: s is perfect square$\}$ Hint: assume $s=n^2$
9. $\{a^c$: c is perfect cube$\}$ Hint: assume $c=n^3$
10. $\{a^{n!} : n \geq 0\}$
11. $\{a^n b^n c^n : n \geq 1\}$ with $\sum = \{a, b,c\}$
12. $\{ww: w \in \{0, 1\}^*\}$ Hint: assume $ww=0^{\mathbf{n}}1^{\mathbf{n}}0^{\mathbf{n}}1^{\mathbf{n}}$ i.e. $w=0^{\mathbf{n}}1^{\mathbf{n}}$
13. $\{ww^R: w \in \{0, 1\}^*\}$ Hint: assume $ww^R =0^{\mathbf{n}}1^{\mathbf{n}}1^{\mathbf{n}}0^{\mathbf{n}}$ i.e. $w=0^{\mathbf{n}}1^{\mathbf{n}}$
14. $\{w \in \{0, 1\}^*$: w has an equal number of 0s and 1s$\}$. Hint: use concept of 0^*1^* and $0^{\mathbf{n}}1^{\mathbf{n}}$
15. $\{w \in \{a, b, c\}^*$: w has an equal number of a, b and c$\}$.
16. $\{a^n b^{2n} c^{3n} : n \geq 1\}$

For the following languages:

a. Write context free grammars (CFG)

b. Design Pushdown Automata (PDA)

1. $\{ a^n b^m : m>n \geq 0 \}$.
2. $\{ww^R: w \in \{a, b\}^*\}$ i.e. each string in L is even palindrome.
3. $\{ w \in \{a, b\}^*\}: w=w^R\}$
4. $\{ w \in \{a, b\}^*\}$: w has equal number of a and b$\}$
5. $\{ w \in \{a, b\}^*\}$: w has twice as many b's as a's$\}$
6. $\{ a^m b^n : m \geq n\}$
7. $\{ w \in \{ (,\ ) \}^*$: each string in w has balanced parentheses$\}$.
8. $\{ a^m b^n c^n : m, n \geq 0 \}$